A Direct Method for Positioning the Arms of a Human Model

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Abstract

Many problems in computer graphics concern the precise positioning of a human figure, and in particular, the positioning of the joints in the upper body as a virtual character performs some action. We explore a new technique for precisely positioning the joints in the arms of a human figure to achieve a desired posture. We focus on an analytic solution for the IK chains of the model's arms and an interface for conveniently specifying a desired targeting point, or articulator, on the model's hand. Also, we consider the problem of specifying a target for that articulator in space or in contact with the model's own body. These methods recast the seven degrees of freedom in the arm to provide a more intuitive interface for animation. We demonstrate the efficacy and efficiency of these techniques in positioning a virtual American Sign Language interpreter.

Key words: Analytic Algorithms, Inverse Kinematics, Human Arm, ASL.

1 Introduction

Many applications in Computer Graphics (CG) require the positioning and animation of articulated figures containing joints with multiple degrees of freedom [1]. In the case of the human body, the animator must coordinate the positioning of dozens of joints. The two arms of a virtual human may contain over 30 joints in the shoulder, elbow, wrist and knuckles [6]. Animators use a range of techniques to manage this complexity, of which the many Inverse Kinematics (IK) methods are among the most widely used [2].

IK techniques were first used in robotics to position a series of joints, so as to place an end-effector on the robot's arm at a position and orientation necessary to perform some task [7]. Compared to robotics, character animation requires finer nuances in a character's motion.

Another application requiring fine nuances in motion is animating American Sign Language (ASL) [6]. ASL is a natural language used by the North American Deaf community and is the fourth most widely used language in North America [8]. The purpose for animating ASL is to support the development of a synthetic interpreter for cases when human translators are unavailable or too expensive [12].

In ASL, subtleties in motion, position and configuration of the arms can make an enormous difference in meaning. One example is the differences between the signs for EYEGLASSES and GALLAUDET UNIVERSITY. See Figure 1 in Appendix A. They both have the same hand configuration (handshape) and both have the same basic movement, but EYEGLASSES happens on the front of the face around the eye, while GALLAUDET happens just to the side of the eye, pulling back towards the ear [10].

There is always a tradeoff between the amount of accuracy and control an application achieves, on one hand, and the speed at which an animator can express their intent, on the other. Applications such as character animation and ASL require computational methods and interfaces that make such fine control easy and intuitive, so that the animator can produce precise and expressive animations.

This paper describes a direct, analytic IK technique that supports an interface allowing animators to transcribe signs in ASL quickly and precisely. This same technique could also be incorporated into any general animation system for specifying arm movements.

2 Description of the Problem

Building a general system for animating ASL requires highly complex and intuitive controls for the model, two goals that are often at odds. In particular, ASL signs often require that the model's hand be in contact with the other hand or some part of the body or face. For more information on the linguistics of ASL, see [5] or [9].

It is imperative that the positions recorded by the animator be precise enough that the model's fingers do not wind up in collision with the model's own body. Moreover, the specific part of the hand contacting the body will vary from sign to sign. In Figure 2 of Appendix A, the sign for IDEA places the tip of the index finger in contact with the face, whereas the sign for CHOCOLATE uses the lower part of the thumb as the point of contact [10]. We define an *articulator* to be a point on the hand used for targeting.

Since we wanted the computational methods to be as efficient and as stable as possible we rejected the more traditional inverse Jacobean and iterative IK methods. Certainly, other applications call for specifying a target position for the wrist in space, but these can be handled by current analytic IK methods [3] [4]. However, for our application, in addition to the simple case, we also needed to

1. Allow the user to specify several key global orientations for the wrist (such as up, down, in, out, etc.) that are specified at times in ASL grammar, both in spatial positioning and in contact situations.
2. Specify a target for an articulator on the hand, while giving the user complete control over the local rotation at the wrist joint, and then, allow the user to manipulate the local wrist orientation without affecting the position of the articulator.

Problem 1 can be handled quite easily with an application of the techniques in [4]. But an extension of these techniques is necessary to solve problem 2. Tolani, Goswami and Badler [11] considered a problem related to 2, but stopped short of a full direct solution, relying instead on optimization techniques to calculate the elbow's bend angle.

The key difference between problems 1 and 2 is how the user specifies the orientation of the wrist. Case 1 is identical to the grasping task problem encountered in robotics and ergonomics studies. In many grasping tasks the orientation of the palm is specified relative to the object being grasped, i.e. in world coordinates [4]. Given this orientation, the solution is a simple matter of subtracting a vector aligned along the palm to calculate where the wrist must be placed. After this, one can solve the triangle formed by the shoulder, elbow and wrist to get the complete orientation of the arm, see Figure 1.

In animating ASL signs, the primary focus is slightly different, as specified in case 2. Of primary importance is how the wrist and shoulder look in relationship to the rest of the body. Neither joint must look unduly strained. To make such subtle relationships easy to achieve, an animator must be given direct control over the orientation of the wrist relative to the forearm. For the purposes of the IK calculation, we consider the wrist as a fixed rotation relative to the forearm and calculate rotations for the shoulder and elbow necessary to place the articulator in the desired position.

This is similar to the aiming task considered in [11], the key difference being that there they do not specify the distance to the target point. As previously mentioned, their technique requires iterative optimization to calculate the elbow's bend angle in the most general case. However, it turns out that an extension of Kondo's geometric method solves our problem completely and directly.

3 Analytic IK Solutions

Two popular IK methods are the inverse Jacobean and optimization approaches, each of which requires the calculation of a series of approximations converging to the desired solution. Such algorithms are quite effective for general IK problems, but when confronted with a simpler problem such as the orientation of the arm, with only three joints, a more stable, direct solution may be achieved depending on the specific problem and constraints. For more information, [11] has a nice overview of classical analytic and numerical algorithms for IK.

One of the key problems with iterative solutions is their unpredictability and instability in the presence of an underdetermined system, such as the human arm. Consider that, even when placing the wrist at a desired point in space, there are an infinite number of solutions parameterized by the rotation of the system about an axis through the shoulder and wrist, see Figure 1.

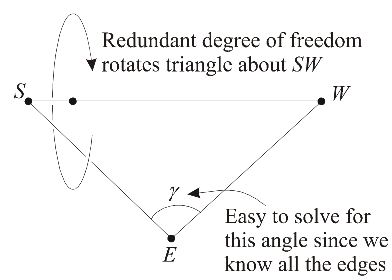

Figure 1: Triangle for Placing Wrist

In a system for positioning a human model, the animator should be given an intuitive set of controls for choosing a desired configuration amongst all of the available solutions. This is not provided by general iterative methods, which most often give unpredictable results for redundant degrees of freedom.

Another key problem is that such techniques can be highly unstable near targets where the iterative solution is ill conditioned. For example, when the Jacobean matrix fails to have full rank or near targets for which there is no solution, iterative methods can become highly unstable, causing the system to fluctuate wildly and never reach an optimal solution [11].

Lastly, iterative methods are computationally inefficient compared to analytic solutions. Therefore, we desired a direct analytic solution that was stable, easy to control and which would allow the user to explore the redundant degrees of freedom in an intuitive manner.

As mentioned previously, there have been several efforts in this direction, for analytic solutions including

1. Traditional IK chains where the shoulder and elbow joints control the positioning of the wrist and the palm is simply an end-effector which will be placed in a desired orientation. It does not affect the IK chain unless the desired position would violate a rotational constraint [2].
2. Grasping tasks where the global orientation of the end-effector is known, and therefore, also is the wrist position [4]. Solving this problem requires solving a triangle, whereas when specifying a *local* orientation of the wrist, the solution requires calculating the angles of a general tetrahedron, as we shall see momentarily.

We achieved our goals by extending the methods of Kondo to place an arbitrary point on the hand at a given position, and given parameters for the redundant degrees of freedom defined by the animator. In addition, the redundant degrees of freedom correspond to intuitive motions of the shoulder and wrist.

4 Our Solution

Consider the IK chain displayed in Figure 2 representing the human arm. This chain has three joints: the shoulder *S*, the elbow *E* and the wrist *W*. The articulator *A* lies on the hand, but does not necessarily lie on the central axis of the hand, as shown in the Figure. This must be taken into account in our calculation, but does little more than add a fixed rotation into the kinematic chain. The articulator may be placed anywhere on the hand, or at the wrist, which then reduces to Kondo's case.

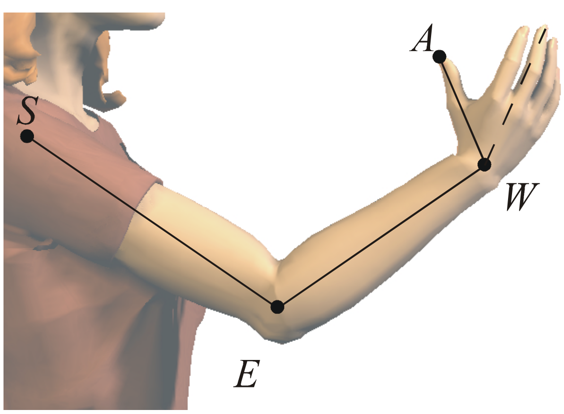

Figure 2: The IK Chain for the Human Arm

The shoulder *S* is a ball joint with 3 degrees of rotational freedom including a radial twist. The elbow is a hinge joint with only one rotational direction, while the wrist has two degrees of freedom, flexion/extension, and abduction/adduction. The radial twist of the forearm-wrist complex happens as the two forearm bones, the radius and ulna, rotate with respect to each other. In our model, we actually place this rotation at the elbow to facilitate deformations in the forearm mesh, but for this discussion we will place that radial twist at the wrist.

When considering the placement of the articulator *A* at an arbitrary target point *P* in space, our system has several redundant degrees of freedom. The first is a rotation ψ of the arm about a line from the shoulder through the articulator *A*. *S*ee **Figure 3**. The other degrees we will discuss later. For now, we will assume that we have chosen a fixed orientation *R* for the wrist. With the choices of *R* and ψ, the system is no longer underdetermined.

Thus, given an orientation *R* of the wrist, and a chosen rotation ψ of the system about the shoulder-articulator axis, *SA*, we wish to calculate the rotations of the shoulder and elbow that will place the articulator *A* at the chosen target *P*. We will represent the orientation of the upper arm *SE* in spherical coordinates, which can be converted from there into Euler angles if the application so dictates. Also, we will initially calculate the orientation of the system in a chosen default orientation of $\psi = 0$.

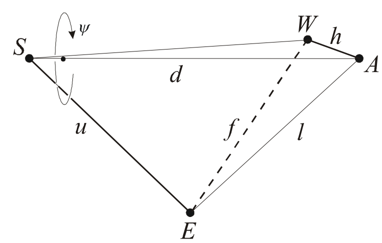

Figure 3: The Tetrahedron Formed by the Arm

To facilitate the definition of a spherical coordinate system for the upper arm, define a local coordinate system with origin at *S*, and with *z*-axis pointing towards the negative vertical in the world system. This choice is compatible with the physiology of the arm because the rest position of the arm places the elbow and wrist below the shoulder in a vertical line against the body. From here, we define two spherical angles ϕ_s, θ_s as shown in Figure 4. The third degree of freedom for the shoulder is then the radial twist τ_s, which doesn't enter this calculation until we consider the rotation ψ.

The solution follows in several steps by basic trigonometry, since the points in this system define a tetrahedron formed by the convex hull of the four points {*S, E, W, A*}. Figure 3 displays this tetrahedron with the original arm chain in bold. The solution of this problem amounts to solving for the dimensions and orientation of this tetrahedron in space. The main degenerate case we need to worry about is if the tetrahedron collapses to a line.

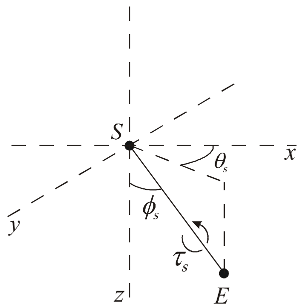

Figure 4: The Shoulder Coordinate System

4.1 Calculating the Elbow Bend Angle

Since we are given the dimensions of the arm's components, the fixed position of A relative to W, and the desired location of the articulator, $A = P$, we know the following five lengths

$$u = |SE| \qquad f = |EW|$$

$$h = |WA| \qquad d = |SA|$$

$$l = |EA|$$

From now on, we will not distinguish between A and P. Since we know all sides of the front triangle SEA, we can solve for its angles with the law of cosines

$$\alpha = \arccos\left(\frac{u^2 + l^2 - d^2}{2ul}\right)$$

Note that we are not given the position of W, nor do we yet know the orientation of the tetrahedron, so we do not know the global direction of the vector $A - W$. More importantly, we do not know the key angle $\gamma = SEW$, shown in Figure 5, which is the desired elbow rotation.

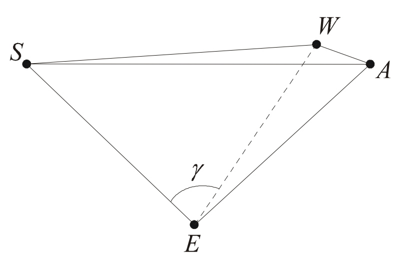

Figure 5: The elbow rotation γ

To find γ we must first reorient the tetrahedron to facilitate certain calculations. Rotate the tetrahedron so that EW lies on the vertical z-axis and S lies in the positive x portion of the x,z-coordinate plane, as indicated in Figure 6. Thus we are looking at the tetrahedron in the coordinate system of the elbow joint, a system that allows us to effectively leverage the given information.

We are given the fixed orientation of the wrist and position of the articulator. Combining this into a single transformation, we can calculate the coordinates of A in the local coordinate system of the elbow

$$A = (h\cos\theta_w \sin\phi_w, h\sin\theta_w \sin\phi_w, h\cos\phi_w + f)$$
$$= (x, y, z)$$

where θ_ω, ϕ_ω are the spherical coordinates of A with respect to the wrist, and h is the length of the hand. Let $A' = (x, 0, z)$ be the projection of A to the x,z-plane, which means that the angle $\varepsilon = \angle$ WEA' = arctan(x/z).

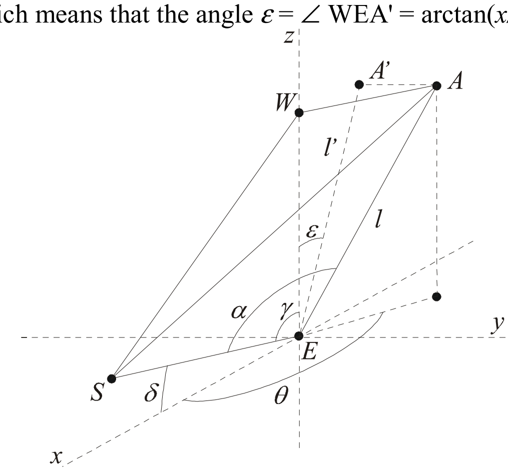

Figure 6: Solving for γ

Let δ be the angle that SE makes with the x-axis, and notice that the desired elbow angle $\gamma = \pi/2 - \delta$. So, we solve this problem by calculating δ. To this end, notice that whatever δ is, we have the following relationships by trivially rewriting the given lengths.

$$|SE| = u = \sqrt{u^2\cos^2\delta + u^2\sin^2\delta}$$

$$|SA| = d = \sqrt{(u\cos\delta - x)^2 + (u\sin\delta - z)^2 + y^2}$$

$$|EA| = l = \sqrt{x^2 + y^2 + z^2}$$

Now using the expression for α given by the law of cosines, and a few trigonometric manipulations

$$\cos\alpha = \frac{u^2 + x^2 + y^2 + z^2 - (u\cos\delta - x)^2 - y^2 - (u\sin\delta - z)^2}{2u\sqrt{x^2 + y^2 + z^2}}$$

$$= \frac{u^2 - u^2(\cos^2\delta + \sin^2\delta) + 2u(x\cos\delta + z\sin\delta)}{2ul}$$

$$= \frac{x\cos\delta + z\sin\delta}{l}$$

We can use the expression which calculated ε to write x

and y in terms of ε. Let l' be the length of EA' as shown in the figure, then

$$A' = (x, 0, z) = (-l' \sin\varepsilon, 0, l' \cos\varepsilon)$$

and therefore, by the angle difference identity for sin

$$\cos\alpha = \frac{l' \sin\delta \cos\varepsilon - l' \cos\delta \sin\varepsilon}{l}$$
$$= \frac{l'}{l} \sin(\delta - \varepsilon)$$

which then can be rewritten as

$$\delta = \arcsin\left(\frac{l}{l'} \cos\alpha\right) + \varepsilon$$

Since $\gamma = \pi / 2 - \delta$, this completes the construction of the elbow's bend angle. Note, however, that this calculation can fail in one of two situations.

1. If $l' = 0$, then the orientation of the wrist combined with the location of the articulator relative to the wrist has placed A at the elbow. Since this is impossible given the physiology of a normal human arm, we ignore this case.
2. Second, is the possibility that

$$\text{abs}\left(\frac{l}{l'} \cos\alpha\right) > 1$$

   This happens when the target is unreachable, and if we clamp this value to 1, we will get the closest attainable position for A.

4.2 Calculating the Shoulder Orientation

At this point we have calculated the angles of the tetrahedron. From here, we just need to calculate the orientation of this tetrahedron in space to determine the necessary orientation of the shoulder joint S. Note that we have two fixed points on the tetrahedron: S and A. So, the only degree of freedom we have left is that of rotating the tetrahedron about the edge SA. This angle is the control angle ψ specified in the statement of the problem.

Remember, that we begin by finding the orientation of the tetrahedron in a standard position where $\psi = 0$, and then rotate the system by ψ about SA from there. As a default reference orientation, we have three possibilities:

1. If S, E and W are not collinear, then we orient the tetrahedron so that S, E and W are all in the vertical plane formed by S, W and the z-axis. See **Figure 7**.
2. If $\gamma = \pi$ or $\gamma = 0$, meaning that S, E and W are collinear, then we set the default orientation to be when S, W, E and A are all coplanar with the z-axis.
3. If all four points are collinear, then the system is completely independent of ψ and so the system is already completely determined, and we just orient the shoulder to the same spherical coordinates, relative to S, as A itself.

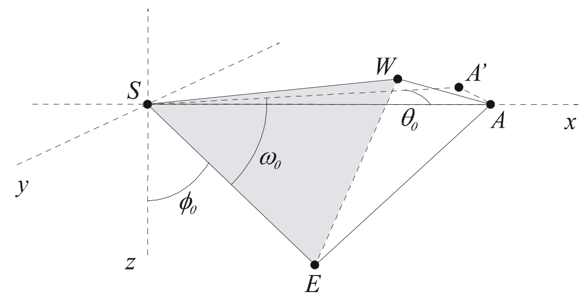

Figure 7: The Spherical Coordinates of SE in the Default Position

Also, we make the simplifying assumption that A lies along the x-axis in the shoulder's coordinate system so that the spherical orientation of A relative to S becomes $\phi_A = \pi / 2$, $\theta_A = 0$. Once we find the orientation of the shoulder, (ϕ_0, θ_0) in this default case, we can simply add the spherical orientation of A to that of SE to obtain the final orientation: ϕ_s, θ_s.

The setup of this calculation is displayed in **Figure 7**. Notice that W may be above or below the x,y-plane, but that E lies directly below the line SW. As indicated in the figure, ϕ_0 is the angle formed by the z-axis and SE, which is $\pi / 2 - \omega_0$, as shown. To obtain these angles, we appeal, once again, to the projection A' of A to the plane formed by SEW. Thus,

$$\phi_0 = \pi / 2 - \arccos\left(\frac{SE \bullet SA'}{|SE||SA'|}\right)$$
$$\theta_0 = \arccos\left(\frac{SA \bullet SA'}{|SA||SA'|}\right)$$

The only problem being that we don't know the position of E, or the position of W in this base coordinate system. But we do know their positions in the elbow's coordinate system, displayed in Figure 8. In this coordinate system,

$$S = (u \cos\delta, 0, u \sin\delta)$$
$$E = (0, 0, 0)$$
$$A = (x, y, z)$$
$$A' = (x, 0, z)$$

Since we just calculated δ in the last section, we know the positions of each of these points, and so we can calculate all of the vectors required to determine ϕ_0, θ_0. Then if ϕ_A, θ_A are the spherical angles of A in the base coordinate system, we finish the construction by setting

$$\theta_S = \theta_0 + \theta_A$$
$$\phi_S = \phi_0 + \phi_A$$

Notice that this part of the construction will fail if

1. $|SA| = 0$, in which case the target is at the shoulder. Thus, the tetrahedron collapses to a triangle and simpler methods may be used. Physiologically, it is painful to place part of the hand in the center of the shoulder joint.
2. $|SA'| = 0$, in which case the target and the wrist lie on perpendicular axes through the shoulder. An example would be if the wrist lies on the x-axis in the shoulder's coordinate system and the wrist lies in the y,z-plane. Thus $\phi_0 = \pm\pi/2$, depending on the orientation of A with respect to W. This corresponds to a physical action of placing your wrist directly in front of your shoulder and the articulator on the line coming horizontally out of the shoulder. This position would certainly put strain on both the shoulder and the wrist.

As long as we take care of these two exceptional cases, the construction is complete. Note that the entire construction is analytic and completely determined up to four redundant degrees of freedom, which we leave at the control of the user.

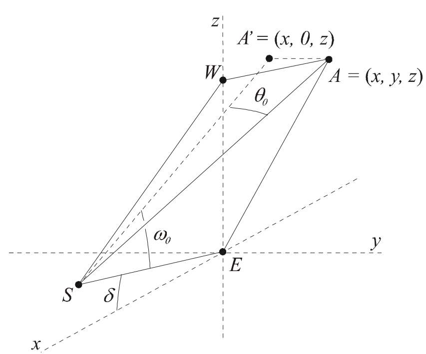

Figure 8: Angles in the Elbow's Coordinate System

4.3 Degrees of Freedom

In our complete kinematic chain for the human arm, there are a total of seven degrees of rotational freedom: three angles in the shoulder, one in the elbow, and three in the wrist. Positioning the articulator gives the animator three degrees of freedom for working with the model. There are four more degrees of freedom in the model, which the above algorithm conveniently parameterizes for the animator.

We have already discussed the fact that the rotation of the system about the axis through the shoulder and articulator is a redundant degree of freedom that the animator may use to achieve a desired posture.

The rotation of the wrist provides the remaining three degrees. If the user changes the wrist orientation, but leaves the articulator target fixed, then the above algorithm can adjust the rest of the angles in the system to compensate, leaving the articulator position unchanged. This compound operation forms three degrees of freedom for the system parameterized by the three angles of rotation for the wrist. Note the difference between this and the usual forward kinematic treatment of wrist rotations.

Thus, the user has full control over an intuitive parameterization of the seven degrees of freedom in this model, including the position of the articulator.

5 Applying the Technique

We applied this technique in a general system for transcribing ASL signs. First consider the positioning of the articulator. This action forms the first three degrees of freedom described above.

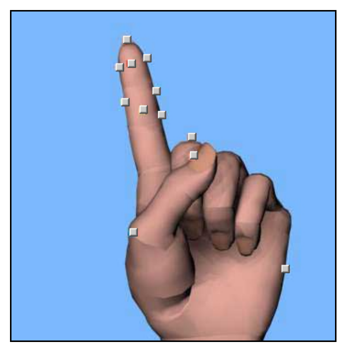

Figure 9: Choosing Articulators on a Handshape

In ASL, each specific configuration of the hand, or handshape has a number of different legal articulators defined by ASL linguistics. See Figure 9. Also, each articulator may be placed in contact with various points on the body, the face or the other hand**.** This works for positioning either hand. Figure 10 shows the interface for choosing such sites on the face. These sites are also predetermined by the linguistics of ASL. Our system also contains a method for specifying a spatial position when the articulator is not in contact with the body.

Once a target (either spatial or contact) has been chosen, the user is free to adjust the orientation of the wrist and the height of the elbow (our angle ψ) with the control panel displayed in Figure 3 of Appendix A. This figure displays the results obtained by flexing the wrist when the tip of the index finger is in contact with the model's cheek.

As the user is adjusting these controls, the above IK method automatically adjusts the orientation of the shoulder and elbow so that the articulator stays in place. This eliminates a process of successive refinements that would be necessary if the system used an ordinary IK chain where the wrist is set as an end-effector. The user would have to successively rotate the wrist and then adjust the wrist's position to compensate, an operation automated by our IK technique.

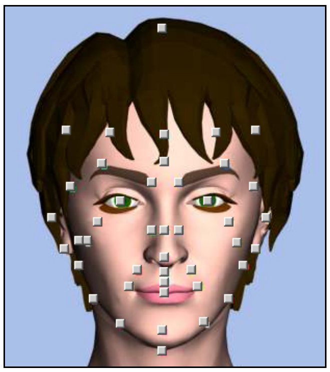

Figure 10: Choosing Targets on the Face

6 Conclusion and Future Work

The calculations of the shoulder and elbow angles θ_s, ϕ_s and ψ, detailed in section 4.2, provide an analytic parameterization of the seven degrees of freedom present in the wrist and arm of the human body. Using these parameters, we obtain an intuitive set of controls for choosing key positions for the arms of a human model. These key positions can include hand/body contact, and can be easily interpolated to display animations of the character's motion.

We are currently integrating this system into a complete model of the upper body, including the spine, neck and collarbone. In particular, the model described in this paper assumes that the shoulders are fixed with respect to the torso, which is simply not true in a human model. Nor is the motion of the shoulders a trivial one. It accounts for much of the expressiveness of the upper body especially for motions like shrugging and slumping. A complete, expressive human model must include such motions.

To make the animator's job easier we are currently working to create an automatic algorithm to coordinate some of the shoulder's motion with the arm's reach. For example, when the model reaches for a target far from the body, the shoulder should automatically move forward to extend the reach of the model.

References

[1] Badler N, Phillips C, Webber B. Simulating Humans: Computer Graphics Animation and Control. *Oxford University Press*. New York, NY. 1993.

[2] Girard M, Maciejewski A. Computational Modeling for the Computer Animation of Legged Figures. *Computer Graphics*. 19(3), July 1985, Pages 253-270.

[3] Koga Y, Kondo K, Kuffner J, and Latombe J. "Planning Motions with Intentions," *Proc., SIGGRAPH'94,* Orlando, FL, July 24-29, 1994. Pages 395-407.

[4] Kondo K. Inverse Kinematics of a Human Arm. *Technical Report STAN-CSTR-94-1508*, Dept. of Computer Science, Stanford University.

[5] Liddel S, Johnson R. American Sign Language: The Phonological Base. *Sign Language Studies.* 64, 1989, Pages 195-277.

[6] McDonald J, Toro J, et. al. An Improved Articulated Model of the Human Hand. *Proceedings of the 8th International Conference in Central Europe on Computer Graphics, Visualization and Interactive Digital Media*. 2000. Pages 306 – 313.

[7] Murray R, Li Z, and Sastry S. A Mathematical Introduction to Robotic Manipulation. *CRC Press, Inc.*, 1994.

[8] National Institute on Deafness and other Communication Disorders, Website. Current as of March 2002. http://www.nidcd.nih.gov/health/pubs_hb/asl.htm.

[9] Sandler W. Phonological Representations of the Sign: Linearity and Nonlinearity in American Sign Language. *Foris Publications*, Dordrecht, Holland. 1989.

[10] Tennant R, Brown M, Nelson-Metlay V, The American Sign Language Handshape Dictionary, *Clerk Books*, Washington, DC, 1998, pages 74-75.

[11] Tolani D, Goswami A, and Badler N. Real-time inverse kinematics techniques for anthropomorphic limbs. *Graphical Models* 62 (5), September 2000, Pages 353-388.

[12] Wolfe R, et al. An Interface for Transcribing American Sign Language. SIGGRAPH 99 Sketches, August 11, 1999, Page 229.

Appendix A: Signs and Controls

Figure 1: Sign for GLASSES (left two frames) and GALLAUDET UNIVERSITY (right two frames)

Figure 2: Signs for IDEA (left) and CHOCOLATE (right)

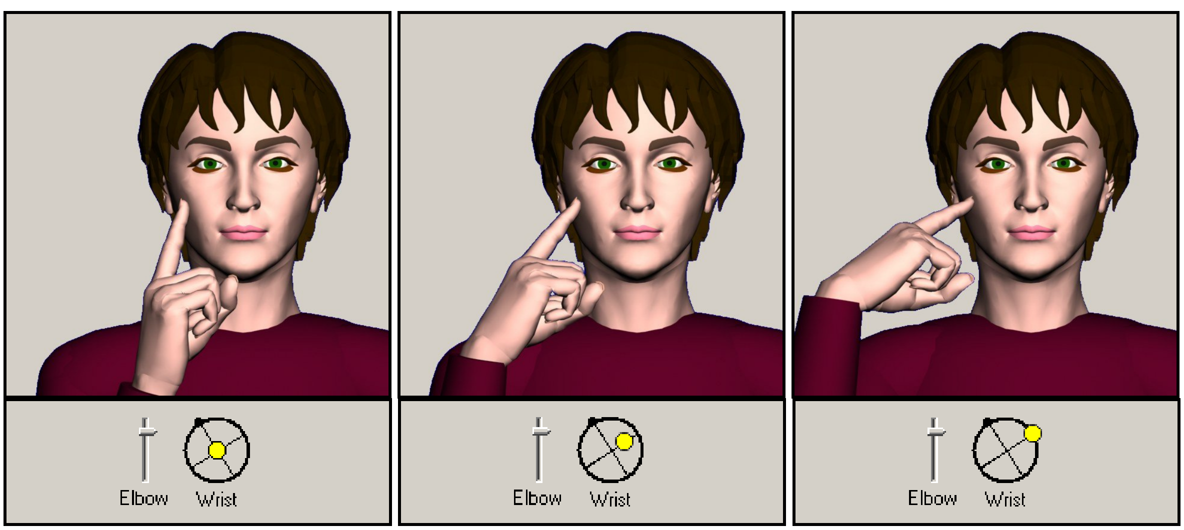

Figure 3: Moving the Wrist Controls with an Articulator in Contact with the Face